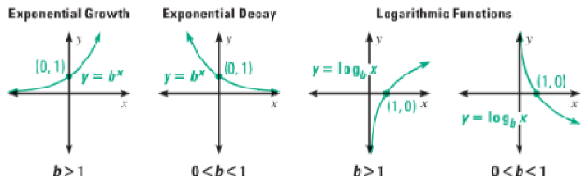
Graphing Exponential and Logarithmic Functions

Chapter 7 Algebra 2 Study Guide

Parent functions for exponential functions have the form $y = b^x$. Parent functions for logarithmic functions have the form $y = \log_b x$.



D: Real | D: Real | D: $x > 0$ | D: $x > 0$

R: $y > 0$ | R: $y > 0$ | R: Real | R: Real

Always on the parent graph: (0 , 1) and (1 , b)

Always on the parent graph: (1 , 0) and (b , 1)

Exponential: $y = b^{(x-h)} + k$
Horizontal Asymptote at k.

Horizontal shift: *h*
Vertical shift: *k*

Logarithmic: $y = \log_b (x - h) + k$
Vertical Asymptote at h.

Compound Interest

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by this equation:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuously Compounded Interest

When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

where P is the principal and r is the annual interest rate expressed as a decimal.

Definition of Logarithm with Base *b*

Let b and y be positive numbers with $b \neq 1$. The **logarithm of *y* with base *b*** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

Examples: $\log_3 81 = 4$ | $5^4 = 625$ | $\ln 6 = 1.792$ | $\log 1000 = 3$

$3^4 = 81$ | $\log_5 625 = 4$ | $e^{1.792} = 6$ | $10^3 = 1000$

Properties of Logarithms

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Expand: $\log 8x^3$

$\log 8 + \log x^3$

$\log 8 + 3 \log x$

Condense: $5 \log_4 5 + 2 \log_4 x$

$\log_4 5^5 + \log_4 x^2$

$\log_4 5^5 \cdot x^2$

$\log_4 3125 x^2$

Solving Exponential and Logarithmic Equations

Solving an Exponential Equation	Solving a Logarithmic Equation
If each side can be written using the same base, equate exponents. $3^{x+1} = 9^x$, $3^{x+1} = (3^2)^x$, $x + 1 = 2x$, $1 = x$	If the equation has the form $\log_b x = \log_b y$, use the fact that $x = y$. $\log_2 (4x - 2) = \log_2 3x$, $4x - 2 = 3x$, $x = 2$
If each side cannot be written using the same base, take a logarithm of each side. $6^x = 15$, $\log_6 6^x = \log_6 15$, $x = \frac{\log 15}{\log 6} \approx 1.511$	If a logarithm is set equal to a constant, exponentiate each side. $\log_5 (x + 1) = 2$, $x + 1 = 5^2$, $x = 24$

Change-of-Base Formula

If a, b, and c are positive numbers with $b \neq 1$ and $c \neq 1$, then:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Evaluate: $\log_5 20$

$$\frac{\log 20}{\log 5} = 1.86$$

$$5^{1.86} = 20$$

Inverses:

Switch x and y.
Isolate the log or exponent term.
Write in other format. (log ⇔exponential)
Solve for y.

Find the inverse of $y = 2^x + 3$

$$x = 2^y + 3$$
$$x - 3 = 2^y$$
$$\log_2(x-3) = \log_2 2^y$$
$$\log_2(x-3) = y$$

Find the inverse of $y = \ln(x - 4) + 2$

$$x = \ln(y-4) + 2$$
$$x - 2 = \ln(y-4)$$
$$e^{x-2} = e^{\ln(y-4)}$$
$$e^{x-2} = y - 4$$
$$e^{x-2} + 4 = y$$

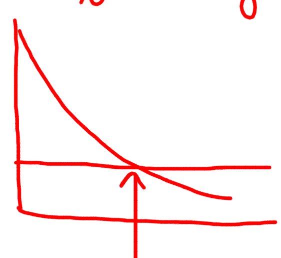
Applications of Exponential Growth and Decay

Harry has 90,000 hairs on his head but he is going bald. Each year he loses 15% of his hairs. Write an equation to model the number of hairs on his head and predict when he will have fewer than 10,000 hairs left.

Jimmy-Joe-Bill-Bob has a worm farm. Right now he has 500 worms, but they are reproducing at a rate of 9% each month. Write an equation to model the worm population and predict the number of worms 9 months from now.

$$y = 90{,}000(.85)^x$$
$$10{,}000 = 90{,}000(.85)^x$$

$x = 13.5$ years



13.5 years

$$y = 500(1.09)^x$$
$$y = 500(1.09)^9$$
$y = 1086$ worms